which indicates that the intensity measurement is independent of RF and modulating signal levels.

With an IF substitution technique, k would be adjusted to satisfy (12.2) while α would be set to unity. However, accurate IF substitution measurements require mixers whose conversion loss is constant over the dynamic range of RF input levels. The RF substitution method was therefore chosen for intensity measurement because a constant power level is maintained at the mixers since k is fixed and α is varied by the servomechanism.

Although complete carrier suppression is not obtained ($\eta \neq 0$), the accuracy of intensity measurement is not seriously limited because $\alpha\alpha_z$ now depends on the ratio of V to V_r and not on their product, and the former may be taken to be substantially constant.

The phasing error δ in (12.1) arises from incorrect balance in the phase measurement and phase shift variations in the RF attenuator. It amounts to about 6°, which represents an error in intensity measurement of 0.05 dB. Other sources of error in recorded intensity arise from: level variations in the homodyne carrier channel (less than 0.2 dB); from the reference attenuator; from the nonlinear potentiometer which translates the attenuator's shaft position to a direct voltage which is applied to a strip chart recorder; from the recorder itself; and from the resolution limit imposed by the error threshold required to operate the servomechanism.

Typically, 20-dB differences may be recorded to an accuracy of 0.6-dB and 40-dB differences to an accuracy of 1 dB.

V. Assessment of the System

The system described is not expensive when compared with alternative automatic systems using IF measurement techniques. It may be constructed from microwave components usually available in a laboratory and imposes no special requirements in the design of the mixers or the low-frequency amplifiers.

Two systems based on these principles have been constructed, operating at 3.20 cm (9.375 GHz) and at 1.276 cm (23.5 GHz), respectively, and a third system for use at 0.4 cm (75.0 GHz) is under development.

One drawback of this measurement system is the response time of the servomechanisms, which can introduce appreciable errors when scanning fields in the vicinity of a null. For example, the recorded 180° phase jump [4], [5] at a null is not infinitely sharp, but may appear to be spread over distances ranging from 0.05 λ to 0.25 λ, depending on the velocity of the scanner drive. For similar reasons, the recorded intensity will be in error on the high side when the probe moves into a null field region and on the low side when the probe moves out. All of these problems may be circumvented, of course, by resorting to point-by-point measurements in regions where the field changes very rapidly.

These apparatus have been extensively used for exploratory and corroborative measurements in microwave diffraction. In such studies, the wavefront incident on the diffracting obstacle must be accurately known and in this regard it has been found that the best results are obtained by placing an open-ended waveguide as far from the obstacle as is practicable. Placing the obstacle 2 m away from the waveguide in the K-band system, for instance, results in nearly spherical wave illumination. Over a cross section 0.65-m square, the measured phase of the incident field differs by about the measurement error (3°–6°) from values for a spherical wave.

A source of error whose magnitude is difficult to assess is the transfer characteristic of the probe used to measure the field. Interaction between the probe and field is only significant near the boundaries of scatterers [3] where standing waves can be set up between them and the probe. However, any practical probe will not respond to the field at a point, but rather to some weighted value of the field in its vicinity. The probe which has been found most satisfactory is a slot-fed dipole antenna attached to the end of a 2-mm OD rigid coaxial line which extends approximately 50 cm in front of the absorber mounted on the scanner mechanism (see Fig. 3).

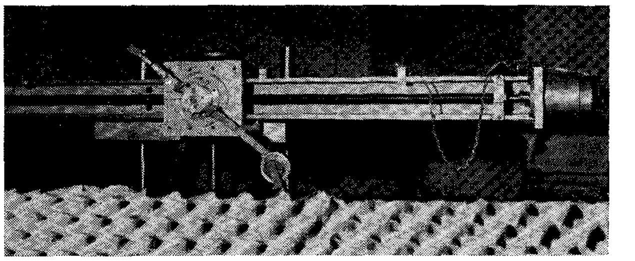

Fig. 3. Probe mount and transverse drive.

Theoretically, such a probe should be linearly polarized and should detect only the field at its "wings." Nevertheless, it appears that the probe is sensitive to cross-polarized components and also to fields extending some millimeters down the supporting coaxial line. However, isolation between the principal and cross-polarized components of better than 45 dB may be achieved [18].

In conclusion, it is the authors' observation, although this is not elaborated in this paper, that errors in the automatic measurement system described are of less concern in near field measurements than uncertainties introduced by reflection from absorbers in the anechoic room; uncertainties in probe characteristics; and errors in the mechanical positioning of the probe.

References

[1] C. L. Andrews, "Diffraction pattern of a circular aperture at short distances," *Phys. Rev.*, vol. 71, pp. 777–786, June 1947.
[2] H. Iams, "Phase-front plotter for centimeter waves," *RCA Rev.*, vol. 8, p. 270, June 1947.
[3] C. L. Andrews, "Diffraction pattern in a circular aperture measured in the microwave region," *J. Appl. Phys.*, vol. 21, pp. 761–767, Aug. 1950.
[4] R. M. Barrett and M. H. Barnes, "Automatic antenna wave-front plotter," *Electronics*, vol. 25, pp. 120–127, Jan. 1952.
[5] M. P. Bachynski and G. Bekefi, "Study of optical diffraction at microwave frequencies," *J. Opt. Soc. Amer.*, vol. 47, pp. 428–438, May 1957.
[6] G. E. Schafer and R. R. Bowman, "A modulated sub-carrier technique of measuring microwave attenuation," *Proc. Inst. Elec. Eng.*, vol. 109, pt. B, supp. 23, pp. 783–786, May 1962.
[7] S. B. Cohn and N. P. Weinhouse, "An automatic microwave phase-measurement system," *Microwave J.*, vol. 7, pp. 49–56, Feb. 1964.
[8] B. B. O'Brien, "A pseudo-superheterodyne receiver for measuring phase and amplitude," *IEEE Trans. Instrum. Meas.*, vol. IM-16, pp. 124–128, June 1967.
[9] S. D. Robertson, "A method of measuring phase at microwave frequencies," *Bell Syst. Tech. J.*, vol. 28, pp. 99–103, Jan. 1949.
[10] G. Bekefi, "Studies in microwave optics," ASTIA Doc. AD110152, ch. 9, (T. J. F. Pavlasek), Mar. 1957.
[11] G. E. Schafer, "A modulated subcarrier technique of measuring microwave phase shifts," *IRE Trans. Instrum.*, vol. I-9, pp. 217–219, Sept. 1960.
[12] R. J. King, "An amplitude and phase measuring system using a small modulated scatterer," *Microwave J.*, vol. 8, p. 51, Mar. 1965.
[13] D. A. Ellerbruch, "Evaluation of a microwave phase measurement system," *Nat. Bur. Stand. J. Res.*, vol. 69C, pp. 55–65, 1965.
[14] R. J. King and R. I. Christopherson, "A homodyne system for the measurement of microwave reflection coefficients," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-18, pp. 658–660, Sept. 1970.
[15] M. M. P. Lehmann, "Measurement of relative phase and intensity of radiated microwave fields of 12 mm. wavelengths," thesis, McGill Univ., Montreal, Que., Canada, 1965.
[16] M. E. Brodwin, C. M. Johnson, and W. M. Waters, "Low level synchronous mixing," *IRE Nat. Conv. Rec.*, vol. 1, pt. 10, pp. 52–57, 1953.
[17] S. Silver, *Microwave Antenna Theory and Design*. New York: Dover, 1965.
[18] J. A. Roth, "A near field study of the principal and cross polarized components of a TE_{11} conical horn," thesis, McGill Univ., Montreal, Que., Canada, 1966.

Open-End and Edge Effect in Microstrip Transmission Lines

O. P. JAIN, V. MAKIOS, and W. J. CHUDOBIAK

***Abstract*—An empirically derived relation is reported which accurately describes the open-end effect in microstrip transmission lines on alumina substrates. This empirical equation was used to**

Manuscript received November 27, 1971; revised February 24, 1972. This work was supported in part by the Canadian Department of Communications under Research Contract OGRO-222.
O. P. Jain and V. Makios are with the Faculty of Engineering, Carleton University, Ottawa, Ont., Canada.
W. J. Chudobiak is with the Department of Communications, Communications Research Centre, Ottawa, Ont., Canada.

describe the edge-correction term in Wheeler's general expression for the characteristic impedance for wide lines. It is shown that with this modification there exists a distinct crossover ($w/h \approx 1$) between the narrow- and wide-line approximations.

I. Empirical Relation for Open-End Effect

The open-end effect in microstrip transmission lines has been considered recently by various authors [1]–[3]. Troughton [1] and Napoli and Hughes [3] reported experimental results, while Farrar and Adams [2] gave a computer method for calculating open-end capacitance. However, analytical or empirical relations convenient for the purpose of microstrip component design have not been reported. It is shown in this short paper that the empirical relation which was derived by Altschuler and Oliner [4, eq. (1)] for open-end strip lines [Fig. 1(a)] may be used to calculate the open-end effect in microstrip lines on alumina substrates by replacing b by $2h$ [Fig. 1(b)] in the expression for the parameter c:

$$\Delta l = \frac{1}{K} \operatorname{arccot} \left[\frac{4c + 2w}{c + 2w} \cot (Kc)\right] \tag{1}$$

where

- Δl apparent increase in length of the center conductor due to fringe field at open end;
- λ wavelength in the medium;
- K $= 2\pi/\lambda$;
- c $= b \ln 2/\pi$.

Altschuler and Oliner [4] demonstrated that for $Kc \leq 0.3$, (1) reduces to

$$\Delta l = c\left\{\frac{c + 2w}{4c + 2w}\right\}$$

which approximates (1) within 3 percent.

The open-end extensions predicted by (1) are plotted in Fig. 2 and are compared with experimental results compiled by the present authors, with experimental results reported by Napoli and Hughes [3], and with the results predicted by the numerical analysis program of Farrar and Adams [2]. An open-end transmission resonator [1] was used by the present authors to obtain the experimental results plotted in Fig. 2. Microstrip lines on thick substrates ($h = 0.0625$ and 0.125 in, $\epsilon_r = 10$, measuring frequency ≈ 2.0 GHz) were used since the resulting open-end extension (Δl) may be measured with much greater accuracy than is possible with thin substrates (say 0.025 in). It is seen that experimental and predicted results are in close agreement (within ± 10 percent).

The curve given by (1) lies, in general, within the experimental tolerances reported by Napoli and Hughes [3] (thin substrates, $h = 0.025$ in), but it is difficult to specify the degree of agreement because of the nature of the tolerances.

The curve given by (1) agrees very closely with that reported by Farrar and Adams [2] for $w/h \lesssim 1$, but the two curves diverge for large w/h ratios since (1) is asymptotic to 0.44 h, while the analysis of Farrar and Adams predicts an inverse relationship between $\Delta l/h$ and w/h for $w/h \gtrsim 6$. As shown in Fig. 2, the experimental results obtained by the present authors suggest that $\Delta l/h$ is asymptotic to a limiting value.

Equation (1) was developed with specific reference to alumina substrates since this is the most commonly used high dielectric constant ($\epsilon_r \approx 10$) substrate material. There is little necessity for such a design equation for low dielectric constant ($\epsilon_r \approx 2.2$) substrates since such substrates are normally very thin (0.010 in) for X-band applications, and preliminary experimental results indicate that the resulting open-end extension is negligible in comparison with the microstrip wavelength ($\Delta l < 0.005\lambda$ at 10 GHz). Very high dielectric constant ($\epsilon_r > 20$) substrate materials are not in common use since the available

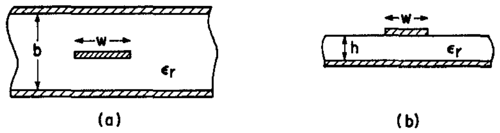

Fig. 1. (a) Stripline geometry. (b) Microstrip geometry.

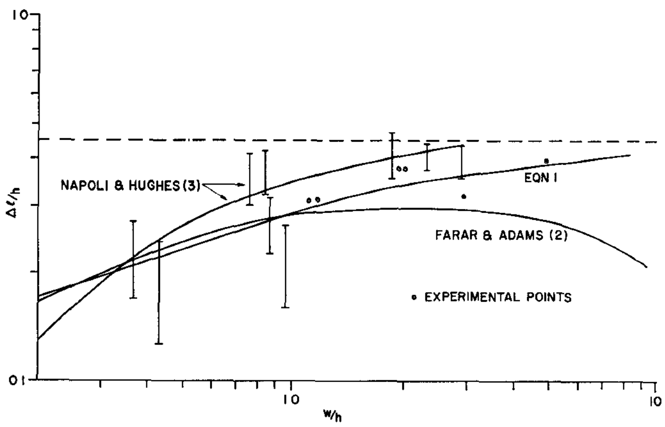

Fig. 2. Measured and predicted values for open-end extensions as a function of the w/h ratio.

materials have high dielectric constant-temperature stability coefficients.

II. A Modification of the Edge Effect for the Wheeler Wide-Line Approximation

At the present time, one of the most convenient and useful techniques for determining the wave impedance and the effective dielectric constant of microstrip lines is that reported by Wheeler [5]. Wheeler derived the following expressions for the wave impedance of wide ($w/h > 1$) and narrow ($w/h < 1$) lines:

$$Z_0 = 0.5 \frac{\left(\frac{1}{\epsilon_r}\right)^{1/2} R_c}{\frac{w}{2h} + c + \frac{\epsilon_r + 1}{2\pi\epsilon_r} \ln \frac{\pi e}{2}\left(\frac{w}{2h} + 0.94\right) + \frac{\epsilon_r - 1}{2\pi\epsilon_r^2} \ln \left(\frac{e\pi^2}{16}\right)} \cdot (w/h > 1) \tag{2}$$

and

$$Z_0 = \frac{R_c}{2}\sqrt{\frac{2}{\epsilon_r + 1}}\left[\ln \frac{8h}{w} + \frac{1}{32}\left(\frac{w}{h}\right)^2 - \frac{1}{2}\frac{\epsilon_r - 1}{\epsilon_r + 1}\left(\ln \frac{\pi}{2} + \frac{1}{\epsilon_r} \ln \frac{4}{\pi}\right)\right] \quad (w/h < 1) \tag{3}$$

where

- R_c wave impedance of free space;
- e $= 2.72$;
- c $= h \ln 4/\pi$, edge correction for an infinitely wide microstrip line.

The effective dielectric constant of the microstrip line may be computed using

$$\epsilon_{\text{eff}} = \left\{\frac{Z_{0a}}{Z_0}\right\}^2 \tag{4}$$

where Z_0 and Z_{0a} are the wave impedances of a microstrip line with and without the dielectric substrate, respectively.

The effective dielectric constant of a microstrip line as a function of the w/h ratio computed using (2)–(4) is plotted in Fig. 3. Curves a (2) and b (3) do not cross, and it is not clear at which w/h ratio one should choose between the narrow- and wide-line expressions. In his equation for the wave impedances of a wide line, Wheeler used the

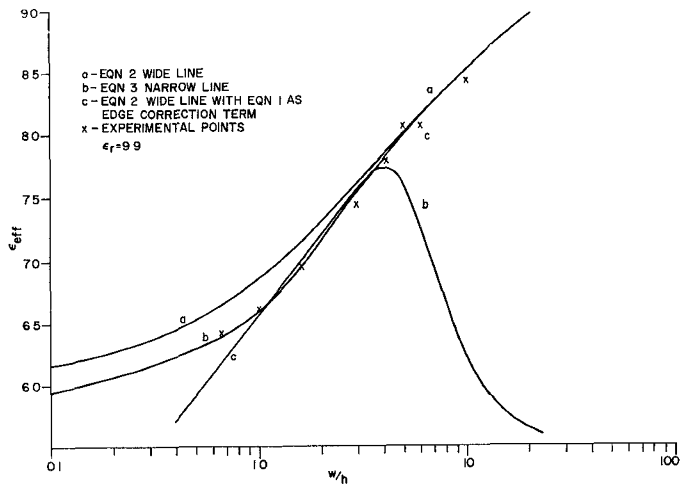

Fig. 3. Microstrip transmission-line effective dielectric constant as a function of the w/h ratio.

expression $(h \ln 4)/\pi$ to account for the edge correction for all values of w/h larger than unity, his reasoning being that the actual edge correction approaches this limiting value very rapidly as w/h approaches and exceeds unity. While attempting to define a crossover point between the narrow- and wide-line cases, the present authors considered the possibility that the edge-correction term for the wide-line case should be a function of the w/h ratio. It was empirically determined that if the edge-correction term in (2) was described in the same fashion as the open-end correction developed in this short paper (1), then a definite crossover point existed between the narrow- and wide-line approximations and the range of validity of the wide-line approximation was extended. Values of ϵ_{eff} given by substituting (1) into (2) are plotted in Fig. 3 (curve c). It is seen that curve c closely approximates the narrow-line curve (curve b) for all values of w/h greater than 1.0 and less than about 3.5 and is asymptotic to curve a for $w/h>3.5$. In order to test the validity of curve c, ϵ_{eff} was measured using a resonant-ring technique for various w/h ratios [6], [7]. The measured results are shown in Fig. 3 and good agreement is exhibited with the modified expression for $w/h>1$ (and with curve b for $w/h<3.5$).

III. Conclusions

An empirically derived relation is reported which characterizes the open-end effects in microstrip transmission lines on alumina substrates. It is shown that the same empirical equation may be used to describe the edge-correction term in Wheeler's general expression for the characteristic impedance of a wide line. It is demonstrated that with this modification, the wide-line approximation is valid over a much larger range of w/h values and that a distinct crossover point ($w/h \approx 1$) exists between the narrow-line and wide-line approximations.

References

[1] P. Troughton, "Design of complex microstrip circuits by measurement and computer modelling," *Proc. Inst. Elec. Eng.*, vol. 118, no. 3/4, pp. 469–474, Mar./Apr. 1971.
[2] A. Farrar and A. T. Adams, "Computation of lumped microstrip capacities by matrix methods—Rectangular sections and end effect," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 495–497, May 1971.
[3] L. S. Napoli and J. J. Hughes, "Foreshortening of microstrip open circuits on alumina substrates," *IEEE Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 559–561, June 1971.
[4] H. M. Altschuler and A. A. Oliner, "Discontinuities in the center conductor of symmetric strip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 328–339, May 1960.
[5] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[6] P. Troughton, "Measurement techniques in microstrip," *Electron. Lett.*, pp. 25–26, Jan. 23, 1969.
[7] O. P. Jain, "A study of dispersive behaviour in microstrip transmission lines," Faculty Eng., Carleton Univ., Ottawa, Ont., Canada, Tech. Rep., May 1971.

Extension of Digital Automatic Method for Measuring the Permittivity of Thin Dielectric Films

M. A. RZEPECKA AND M. A. K. HAMID

Abstract—The permittivity of thin dielectric films can be measured with good accuracy by employing a method recently reported by the authors, whereby the microwave oscillator frequency is automatically locked to the resonant frequency of the test cavity perturbed by the sample, thus leading to a digital readout of the frequency. However, the method is satisfactory only when the frequency shift caused by the presence of the test sample does not exceed the frequency lock-in bandwidth. By employing a search oscillator, controlled by the second harmonic of the modulation signal provided for the frequency locking, this limitation is removed, thus extending the capability of the method to thicker films and/or larger permittivities.

Introduction

The permittivity of thin dielectric films can be measured with considerable accuracy by a digital automatic method recently reported by the authors [1]. The method utilizes the cavity perturbation technique in which the frequency of the microwave oscillator is locked to the resonant frequency of the test cavity in the absence and presence of the test film. These frequencies can be measured very accurately by a digital frequency counter. However, the ranges of film thickness and material permittivity in which the method can be employed are limited since they are determined by the lock-in bandwidth of the frequency control loop. This bandwidth is directly related to the Q factor of the test cavity, which in turn should be large enough to assure a high frequency-stabilization factor, thus permitting only a small deviation of the oscillator frequency from the resonant frequency of the cavity.

In many applications of this measurement method, as for instance in the continuous monitoring of moisture content of sheet materials, the range of the shift in resonance frequency exceeds the lock-in bandwidth of the control loop. A method for extending the measuring range is described in this short paper.

Principle of Operation

The complete circuit is shown in Fig. 1 where the additional parts over the circuit previously reported in [1] are shown by dashed lines. The principle of operation of the locking system is also described in [1]. At the output of the linear homodyne detector there exists a signal at the modulation frequency which is used for the frequency lock, as well as a signal at the second harmonic frequency. Basic signal analysis [1]–[3] shows that the second harmonic signal is described by the equation

$$E(2\omega_m) = -E_{sc}\,|T(\omega)|\,\sin\alpha \sum_{q=1}^{\infty} \frac{r^q p_q'}{q(q+1)}\, G_{q+1,2} \tag{1}$$

where

$$\alpha = \theta - \arg \Gamma$$

$$|T(\omega)| = \frac{T(\omega_0)}{1 + 4Q_L^2 \left(\dfrac{\omega - \omega_0}{\omega_0}\right)^2} \tag{2}$$

Manuscript received December 6, 1971; revised April 14, 1972. This work was supported in part by the National Research Council of Canada under Grant A-3326, in part by the Defence Research Board of Canada under Grants 3801-42 and 6801-37, and in part by the Faculty of Graduate Studies of the University of Manitoba, Winnipeg., Man., Canada.
The authors are with the Department of Electrical Engineering, University of Manitoba, Winnipeg, Man., Canada